Application of the Continuous and Discrete Strategies of Minimum Effort Theory to Interplanetary Guidance

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The theory of minimum effort control is concerned with guiding a vehicle to a specified rms terminal accuracy in the presence of random injection and measurement errors with a minimum amount of expected total velocity correction. This paper applies the theory to guidance problems in typical interplanetary trips. Particular attention is given to a trip to Mars and a flyby trip, Earth-Venus-Mars. Error propagation is obtained from perturbation equations along a nominal Keplerian trajectory, and a "patched-conic" treatment of the nominal trajectory is used for the study of the flyby trips. Orbit determination is assumed to be based on the information obtained from on-board angular measurements as well as Earth-based radar. Computer results are given for typical injection errors indicating the total velocity correction as a function of the required rms accuracy for various information histories. It includes a comparison of the results from a near-optimum strategy using discrete impulsive corrections. A Monte Carlo simulation based on the discrete strategy is performed with the additional feature of being able to control the time of the last correction. Numerical results based on this Monte Carlo program are given showing the advantage of including this additional control as well as the effect of correction mechanization errors.

1. Introduction

THE theory of minimum effort control[1,2] is concerned with the problem of guiding a vehicle to a specified rms terminal accuracy in the presence of random injection and measurement errors with a minimum expected amount of total velocity correction. It is directly applicable to the case of variable time of arrival terminal control assuming that the errors are confined to the transfer plane and that the correction mechanization errors are negligible. The purpose of this paper is to apply this theory to the study of guidance problems in typical interplanetary trips. A computer program is developed which 1) performs a linear error analysis of typical interplanetary trajectories with assumed rms injection errors and measurement histories, and 2) computes a trajectory correction strategy based on minimum effort theory. It includes a near-optimum discrete trajectory correction strategy using impulsive corrections whose spacings are chosen to approximate the ideal continuous strategy. The analysis of these near-optimum discrete strategies extends the study by a Monte Carlo simulation to include the effect of correction mechanization errors as well as the effect of varying the time of the last correction.

Section 2 gives a summary of the theory of minimum effort control. The analogous discrete strategy, including the necessary equations for implementing the Monte Carlo simulation, is given in Sec. 3. Section 4 gives a brief description of the three computer programs that have been developed, and the last section gives the computer results for the velocity correction requirements associated with two typical interplanetary trips. The two trips are: 1) a 204-day trip to Mars, and 2) a 245-day flyby trip, Earth-Venus-Mars.

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2. Summary of the Theory of Minimum Effort Control

2.1 Problem

Let $x(t)$ be a 4 vector consisting of the two position components and the two velocity components of the deviations from a nominal orbit in the plane of the nominal orbit. It will be assumed that the deviations are sufficiently small so that $x(t)$ satisfies a linear differential equation

$$[dx(t)]/dt = F(t)x(t) + f(t) + \eta(t) \tag{1}$$

where $F(t)$ is a 4×4 matrix consisting of the partial derivatives of the equations of motion along the nominal trajectory; $f(t)$ is a 4 vector denoting the applied acceleration and, hence, includes two zero elements $[f_1(t) = f_2(t) = 0]$; $\eta(t)$ is a random 4 vector accounting for the mechanization error. The minimum effort theory assumes that $\eta(t)$ is negligible. Orbit determination is based on a set of noisy measurements. Without loss of generality, it will be assumed that the coordinate system of the measurements has been chosen so that the random errors in the measured components are statistically independent. Then the measurements can be represented by the equation

$$y_i(t) = L_i(t)\, x(t) + \epsilon_i(t) \qquad i = 1, 2, \ldots, r \tag{2}$$

where the scalar $y_i(t)$ represents the deviation of the actual measurement from its nominal value, $L_i(t)$ is a 4 vector of partial derivatives, and $\epsilon_i(t)$ represents the additive random measurement noise. It will be assumed that the measurement noises $\epsilon_i(t)$ are white, Gaussian with zero mean and spectrum

$$\sigma_i^2(t)\,\Delta t_i$$

where σ_i and Δt_i are assumed to be constants and are the rms accuracy of the measurements and the short intervals between successive observations, respectively. Minimum effort theory is concerned with the problem of finding the acceleration vector $f(t)$ as a linear functional of the past observations that minimizes

$$E \int_0^T [f_3^2(t) + f_4^2(t)]^{1/2} dt$$

for specified value of $E[x_1^2(T)]$. In other words, the problem is to control the size of the error in one terminal position with a minimum amount of (expected) total velocity imparted which, in turn, is related to the mass of fuel used for control.

2.2 Preliminary

Let $\hat{x}(t)$ be the best estimate of $x(t)$ based on all the past data

$$\hat{x}(t) = E[x(t)|y_i(s), \quad 0 \leq s \leq t, \quad i = 1, 2, \ldots, r] \tag{3}$$

and let $V(t)$ be the covariance of the estimation error

$$V(t) = \operatorname{cov}[x(t) - \hat{x}(t)] \tag{4}$$

which consists initially of injection velocity error variances. It has been shown[3] that $V(t)$ satisfies the matrix differential equation

$$[dV(t)]/dt = F(t)V(t) + V(t)F'(t) - V(t)\dot{I}(t)V(t) \tag{5}$$

where

$$\dot{I}(t) = \sum_{i=1}^{r} \frac{L_i'(t)L_i(t)}{\sigma_i^2 \,\Delta t_i} \tag{6}$$

is the information rate matrix relative to the state x. Let $D_1(T, t)$ be a 4 vector relating the change in the terminal miss $x_1(T)$ to changes in the state x at time t consisting of both the position and the velocity in the transfer plane. In other words, $D_1(T, t)$ is the first row of the transition matrix $\Phi(T, t) = \partial x(T)/\partial x(t)$ that satisfies the matrix differential equation

$$[d\Phi(T, t)]/dt = -\Phi(T, t)F(t) \qquad \Phi(T, T) = I \tag{7}$$

Define

$$\hat{x}_1(T, t) = D_1(T, t)\hat{x}(t) \tag{8}$$

Then $\hat{x}_1(T, t)$ is the predicted terminal miss based on all the data up to time t. It can be shown that the predicted miss satisfies the stochastic differential equation

$$[d\hat{x}_1(T, t)]/dt = D_1(T, t)\, f(t) + \gamma(t) \tag{9}$$

where

$$\gamma(t) = D_1(T, t)\; V(t) \sum_{i=1}^{r} L_i'(t)\, \frac{[y_i(t) - L_i(t)\hat{x}(t)]}{\sigma_i^2 \,\Delta t_i} \tag{10}$$

and can be considered statistically as a white noise with spectrum

$$b(t) = D_1(T, t)\; V(t)\dot{I}(t)V(t)D_1'(T, t) \tag{11}$$

2.3 Actual Solution

The solution given by minimum effort theory[1, 2] shows that the best linear control is the one in which the acceleration is proportional to the instantaneous predicted terminal miss $\hat{x}_1(T, t)$. The direction of the acceleration is that which maximizes the effect on the terminal miss. In other words,

$$f_3(t) = \frac{-u(t)\phi_{13}(T, t)\hat{x}_1(T, t)}{D_v(t)} \tag{12}$$

and

$$f_4(t) = \frac{-u(t)\phi_{14}(T, t)\hat{x}_1(T, t)}{D_v(t)} \tag{13}$$

where

$$D_v(t) = [\phi_{13}^2(T, t) + \phi_{14}^2(T, t)]^{1/2} \tag{14}$$

is the sensitivity of the terminal miss $x_1(T)$ to velocity changes at time t in the direction of maximum sensitivity. The gain $u(t)$ consists of, in general, three portions: an initial period during which $u(t) = 0$ while the measured information "catches up" with injection errors, followed by a long period during which the gain $u(t)$ varies with time in such a way that the time history of the mean square predicted miss $E[\hat{x}_1^2(T, t)]$ [denoted by $p_{11}(t)$] follows a critical curve p_{11}^* given by

$$p_{11}^*(t) = -[D_v(t)b(t)]/[2\dot{D}_v(t)] \tag{15}$$

and finally, a (short) period of no control at the end. The time t_{on} at which the control is first turned on is determined by

$$p_{11}^*(t_{on}) = \int_0^{t_{on}} b(t)dt \tag{16}$$

and the time at which the control is turned off depends only on the desired mean square terminal miss σ_D^2. Let

$$s_{11}(t) = p_{11}(t) + D_1(T, t)V(t)D_1'(T, t) \tag{17}$$

which is the total mean square terminal error in the absence of any control subsequent to time t. Then t_{off} is determined by

$$s_{11}(t_{off}) = \sigma_D^2 \tag{18}$$

Substituting (12) and (13) into (9) shows that the mean square predicted terminal miss satisfies the differential equation

$$\dot{p}_{11}(t) = -2u(t)D_v(t)p_{11}(t) + b(t) \tag{19}$$

It follows that the average magnitude of the acceleration during the period of continuous control is given by

$$E|f| = \left(\frac{2}{\pi}\right)^{1/2} u(t)[p_{11}^*(t)]^{1/2} = \left(\frac{2}{\pi}\right)^{1/2}\left[\frac{b(t) - \dot{p}_{11}^*(t)}{2D_v(t)[p_{11}^*(t)]^{1/2}}\right] \tag{20}$$

In view of (20), it is seen that the solution is not applicable if the critical curve p_{11}^* has a slope in excess of $b(t)$ at any time. Such is the case, for example, when a sharp increase in information rate is encountered. In this case, minimum effort strategy is to turn off the control some time before this increase and then to turn it on again after some of the new information has been accumulated. From (17), it is seen that $s(t)$ must remain constant while the control is off. Also, it can be shown that the expression $z(t) = D_v^2(t)p_{11}^*(t)$ must have the same value at control cut-off point as at the later control cut-in point. The two timings therefore can be determined by double points in the s-z plane.

3. Near-Optimum Discrete Strategy and Monte Carlo Simulation

Minimum effort theory shows that the optimum linear strategy is continuous. However, practical implementation often requires that corrections be carried out at discrete times. A discrete strategy corresponds to the case in which (20) is replaced by several impulses, and the cumulative effort is composed of a series of upward steps. In other words, we have, for the discrete strategy,

$$u(t) = \sum_{i=1}^{N} g(t_i)\, \delta(t - t_i) \tag{21}$$

where

$$g(t_i) \leq [1/D_v(t_i)] \tag{22}$$

The required velocity correction at t_i is given by $g(t_i)\hat{x}_1(T, t_i^-)$ where $\hat{x}_1(T_1, t_i^-)$ is the predicted miss immediately prior to the correction. It should be noted that equality in (22) implies a "full correction" nullifying the predicted miss at t_i. The determination of a minimum effort N-impulse strategy requires the optimization of $g(t_i)$ as well as the timings of these corrections. This is a tedious problem in ordinary calculus. However, for most purposes, a sufficiently near-optimum discrete strategy can be found by approximating the ideal continuous $s_{11}(t)$ curves with a finite number of downward steps. For the purposes of this study, it will be assumed that the total number as well as the timings of these corrections have been selected in this empirical manner and only the feedback gains $g(t_i)$ actually optimized accordingly. In other words, only the magnitudes of the corrections are optimized.

Let $t_1, t_2, \ldots, t_N$ be the times at which the corrections are applied. The correction times are obtained empirically from the ideal continuous $s_{11}(t)$ curve. If we assume that the last correction must be a full correction, t_N is then determined by the mean square value of the desired terminal miss, namely

$$D_1(T, t_N)V(t_N)D_1'(T, t_N) = \sigma_D^2 \tag{23}$$

Let $p_{11}(t_i^-)$ and $p_{11}(t_i^+)$ be the values of the mean square predicted miss immediately before and after the correction. Substituting (21) into (19) shows that $p_{11}(t_i^-)$ and $p_{11}(t_i^+)$ are related by the difference equation

$$\begin{aligned} p_{11}(t_i^+) &= [1 - g(t_i)D_v(t_i)]^2 p_{11}(t_i^-) \\ p_{11}(t_{i+1}^-) &= p_{11}(t_i^+) + \int_{t_i}^{t_{i+1}} b(t)\, dt \end{aligned} \tag{24}$$

Let $\Delta e(t_i)$ be the required expected velocity correction at t_i so that

$$\Delta e(t_i) = \left(\frac{2}{\pi}\right)^{1/2} g(t_i)[p_{11}(t_i^-)]^{1/2} \tag{25}$$

It is shown in Ref. 4 that the optimum feedback gains $g(t_i)$, and hence the optimum amounts of the N corrections (for fixed t_i) are such that

$$\Delta e(t_i) = \left[\frac{2}{\pi} \cdot p_{11}(t_i^-)\right]^{1/2} \cdot \frac{1}{D_v(t_i)}\left[1 - D_v(t_{i+1})\left(\frac{1}{p_{11}(t_i^-)[D_v^2(t_i) - D_v^2(t_{i+1})]}\right)^{1/2}\right]$$
$$i = 1, 2, \ldots, N-1$$

and

$$\Delta e(t_N) = \frac{1}{D_v(t_N)}\left(\frac{2}{\pi}\, p_{11}(t_N^-)\right)^{1/2} \tag{26}$$

Both the continuous and the discrete minimum effort theories outlined so far assume that the correction mechanization error is negligible. It can be shown[4] that the optimization of the N-impulse strategy as well as of the number N can actually be carried out to include the case of mechanization errors. For the purposes of this paper, this complete optimization will not be attempted. Instead, the effect of random mechanization errors will be studied using the guidance law derived under the assumption that they are negligible. To do this, a Monte Carlo simulation of the discrete strategy outlined in the previous section is developed. The simulation allows the inclusion of mechanization errors in two ways:

1) Engine execution error: The random engine execution error is assumed to be in the direction of the correction. The effect of this error is to increase the variance of the predicted miss. It should be noted that errors in the transverse direction affect only the time of arrival since the orientation has been optimized in the direction of maximum miss distance sensitivity.

2) Accelerometer reading error: This is the error in the knowledge of the actual amount of velocity correction used. This random error causes a loss in information and increases the variances of the estimation errors. It is equivalent to applying impulses at the time of correction to the right-hand side of (5). The effect of these impulses is to cause a jump (or discontinuity) in the elements $v_{ij}(t)$; $i, j = 3, 4$, immediately after the correction.

3.1 Equations for the Monte Carlo Simulation

Let $\Delta e_d(t_i)$ be the amount of the desired velocity correction at t_i, $\Delta e_a(t_i)$ be the actual amount of correction used, and $\Delta e_0(t_i)$ be the reading obtained from the instrument. Then

$$\begin{aligned} \Delta e_a(t_i) &= \Delta e_d(t_i) + \beta(t_i) \\ &= -u(t_i)\hat{x}_1(T, t_i^-) + \beta(t_i) \end{aligned} \tag{27}$$

and

$$\Delta e_0(t_i) = \Delta e_a(t_i) + \alpha(t_i) \tag{28}$$

where $\beta(t_i)$ and $\alpha(t_i)$ are the random engine execution errors and the random reading errors, respectively. It will be assumed that the errors are normal, independent with zero mean and constant variances σ_β^2 and σ_α^2.

It has been stated that the accelerometer reading error causes a loss in information and increases the variances of the estimation errors. We shall now show how this can be imbedded in the integration of the covariance matrix $V(t)$. Let $\Delta\hat{e}(t_i)$§ be the instantaneous conditional mean estimate based on the a priori information $\Delta e_d(t_i)$ and the measurement $\Delta e_0(t_i)$, and let $\Delta\tilde{e}(t_i)$ be the estimation error. Then

$$\Delta\tilde{e}(t_i) = \Delta e_a(t_i) - \Delta\hat{e}(t_i) \tag{29}$$

where

$$\Delta\hat{e}(t_i) = \Delta e_d(t_i) + [\sigma_\beta^2/(\sigma_\beta^2 + \sigma_\alpha^2)][\Delta e_0(t_i) - \Delta e_d(t_i)] \tag{30}$$

Substituting (27, 28, and 30) into (29) yields

$$\sigma_{\Delta\tilde{e}}^2 = \sigma_\alpha^2\sigma_\beta^2/(\sigma_\alpha^2 + \sigma_\beta^2) \tag{31}$$

which, we note, is less than σ_α^2 in view of the given apriori information $\Delta e_d(t_i)$. It can be readily verified that the errors of the estimated state $\tilde{x}$ immediately before and after the trajectory correction are related by

$$\tilde{x}(t_i^+) = \tilde{x}(t_i^-) + \Delta C(t_i) \tag{32}$$

where $\Delta C(t_i)$ is a 4 vector consisting of two zero elements $\Delta C_1 = \Delta C_2 = 0$. The last two elements of $\Delta C(t_i)$ are the components of $\Delta\tilde{e}(t_i)$ given by

$$\Delta C_3(t_i) = \Delta\tilde{e}(t_i)[\phi_{13}(T, t_i)/D_v(t_i)] \tag{33}$$

$$\Delta C_4(t_i) = \Delta\tilde{e}(t_i)[\phi_{14}(T, t_i)/D_v(t_i)] \tag{34}$$

Since $V(t_i) = \mathrm{cov}[\tilde{x}(t_i)]$, it follows that

$$V(t_i^+) = V(t_i^-) + \mathrm{cov}[\Delta C(t_i)] \tag{35}$$

where $\mathrm{cov}[\Delta C(t_i)]$ accounts for the effect of the loss of information and is a 4×4 matrix consisting of all zero elements except the four elements in the lower right corner. This submatrix of nonzero elements is given by

$$\frac{\sigma_{\Delta\tilde{e}}^2}{D_v^2(t_i)}\begin{bmatrix} \phi_{13}^2(T, t_i) & \phi_{13}(T, t_i)\,\phi_{14}(T, t_i) \\ \phi_{13}(T, t_i)\phi_{14}(T, t_i) & \phi_{14}^2(T, t_i) \end{bmatrix} \tag{36}$$

In order to implement the Monte Carlo simulation, we also need the equation governing the instantaneous predicted miss. Using the fact that $\gamma(t)$ may be considered as a white noise and combining Eqs. (9, 11–13) we find

$$\hat{x}_1(T, t_i^+) = \hat{x}_1(T, t_i^-) + D_v(t_i)\,\Delta\hat{e}(t_i) \qquad \hat{x}_1(T;\, 0) = 0 \tag{37}$$

§ This is the instantaneous estimate of the correction used and can, in general, be improved as more data are collected. The latter will require computing the solution of the smoothed estimate[5] and is not considered in this paper.

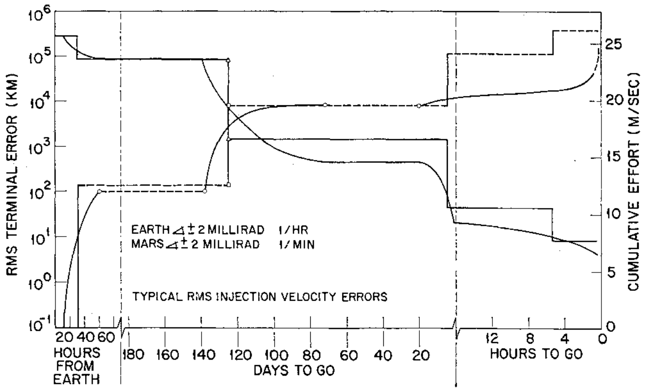

Fig. 1 History of remaining error and cumulative effort vs time to go (204-day trip to Mars).

and

$$\hat{x}_1(T, t_{i+1}^-) = x_1(T, t_i^+) + z_i \tag{38}$$

where z_i is a normal random variable with variance

$$\int_{t_i}^{t_{i+1}} b(t)dt$$

4. Description of the Computer Programs

The main program that performs the linear error analysis and includes the continuous minimum effort correction strategy for typical interplanetary trips consists of two versions: 1) a direct transfer program that considers transfers between two massless planets and 2) a flyby transfer program that considers transfers between two massless planets via a third planet whose gravity field is taken into account. Both versions assume that the errors are confined to the transfer plane.

4.1 Direct Transfer Program

This program assumes that the vehicle is injected from a massless Earth and is transferred to a massless planet. The transition matrices $\Phi(T, t)$, and hence the sensitivities $D_v(t)$, are obtainable from perturbations of a nominal Keplerian trajectory. The program at present allows four kinds of measurements for orbit determination. These are angular measurements of the direction of the sun, the target planet, and the Earth, relative to the star background, and range-rate information from an Earth-based radar. The accuracies and frequencies of these measurements are assumed to be constant in time. The program also has the option of turning on as well as turning off the information from the Earth in the middle of the trip, and turning on the angular information from the target planet. It should be noted that the information from the Earth cannot be turned on immediately, as the formulated information rate is initially infinite.

Other inputs to the program are: 1) the eccentricity, semi-major axis, trip time, and the initial true anomaly; these quantities specify the transfer ellipse by Kepler theory; 2) the initial injection errors in the transfer plane which serve as the initial conditions for the Kalman covariance matrix $V(t)$; and 3) the rms accuracy of the measurements and the corresponding intervals of observation; this and the information in 2) allows us to integrate the equations governing $V(t)$.

One major difficulty in the development of the program was in the integration of the Kalman covariance matrix $V(t)$. It satisfies the matrix Riccati equation (5). The straightforward technique of numerical integration given by

$$V(t + \delta) = \dot{V}(t)\delta$$

was first used. It was found that a sudden large increase in information rate $\dot{I}$ causes numerical instability, and $V(t)$ may then become negative definite. This is circumvented by updating the covariance matrix using the analogous difference equation derived for discrete systems.[3,5] Define

$$V_0(t + \delta) = \Phi(t + \delta, t)V(t)\Phi'(t + \delta, t) \tag{39}$$

which is the updated covariance matrix of the estimation error when no information is taken at $t + \delta$. The inclusion of new information is given by the following equations:

$$V_i(t + \delta) = [I - B_i(t + \delta)L_i(t + \delta)]V_{i-1}(t + \delta) \quad i = 1, 2, \ldots, r \tag{40}$$

where

$$B_i(t + \delta) = \frac{V_{i-1}(t + \delta)L_i'(t + \delta)}{\{L_i(t + \delta)V_{i-1}(t + \delta)L_i(t + \delta) + [(\sigma_i^2\Delta t_i)/\delta]\}} \tag{41}$$

and

$$V(t + \delta) = V_r(t + \delta)$$

It should be noted that no matrix inversion is required in this computation. The program also computes $p_{11}^*(t)$ from (15), t_{on} from (16), the remaining miss $s_{11}(t)$ from (17), and the average effort by integration of (20).

4.2 Flyby Transfer Program

This program extends the program for direct transfer to the case where the transfer is via a third planet. The nominal transfer orbit consists essentially of two heliocentric ellipses connected by a planet-centered hyperbola near the flyby planet. The additional inputs for this program are

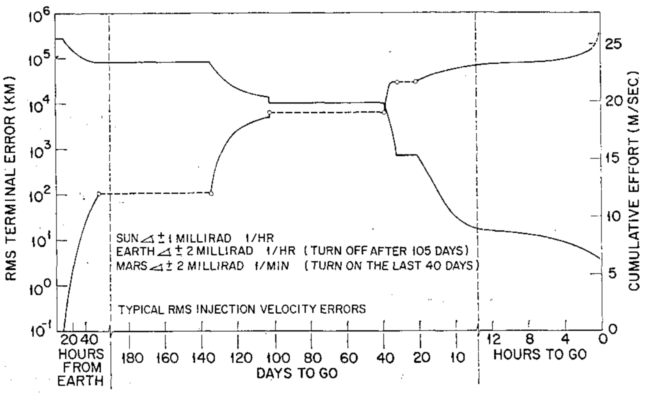

Fig. 2 History of remaining error and cumulative effort vs time to go (204-day trip to Mars).

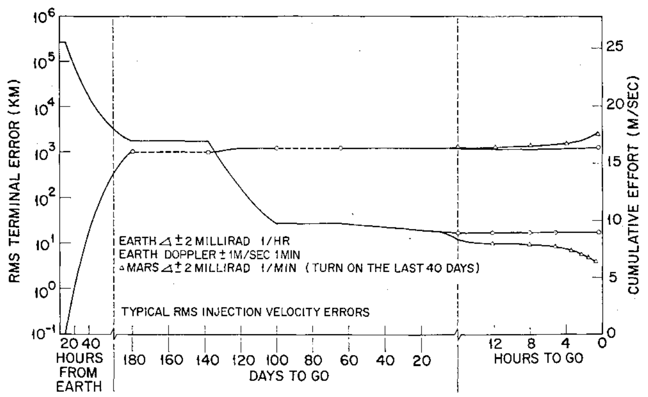

Fig. 3 History of remaining error and cumulative effort vs time to go (204-day trip to Mars).

the necessary parameters which specify the second ellipse as well as the ratio of the mass of the flyby planet to that of the sun. A "patched-conic" treatment of the nominal trajectory is used. The trajectory is taken as a planet-centered hyperbola within a "sphere of influence" near the flyby planet such that the ratio of the sun's "effective" attraction to that of the planet is less than the ratio of the planet's attraction to the (total) attraction of the sun. The boundary of this "sphere of influence" is given by

$$\frac{\text{distance from vehicle to planet}}{\text{distance from planet to sun}} \simeq \left(\frac{\text{mass of planet}}{\text{mass of sun}}\right)^{2/5} \qquad (42)$$

and this is where the two ellipses are patched to the planet-centered hyperbola. The relative velocities of "approach to" and "departure from" the flyby planet must be the same so that the point of closest approach to the flyby planet is halfway along the hyperbolic arc.

4.3 Discrete Minimum Effort Program

This is an additional program which analyzes near-optimum discrete minimum effort control strategies including the effect of mechanization errors. Given the number and the timings of the corrections, the program computes the optimum $g(t_i)$ by using (26). A Monte Carlo simulation based on this discrete strategy is performed.

4.4 Fixed Total Velocity Capability

If we assume a fixed total velocity correction capability somewhat greater than the theoretical expected velocity requirement, the Monte Carlo program has the additional feature of being able to control the time of the last correction so that all remaining fuel is used on this last control, and the estimated miss at this time thereby reduces to zero. In other words, the last correction occurs at the time t_N such that

$$[\hat{x}_1(T, t_N^-)]/[D_v(t_N)] = \text{velocity capability remaining} \qquad (43)$$

t_N being then a random variable. It should be noted that $\hat{x}(T, t)$ for $t \geq t_{N-1}$ is a simple Wiener process.

In the (rare) event that the propulsion left aboard is not sufficient to make the correction called for at an earlier time t_i, $i \leq N-1$, then the program assumes, of course, that the correction uses all the fuel available. This run will then give a terminal miss which is the miss immediately after this correction.

5. Examples and Numerical Results

The programs described in the previous section are used for the study of guidance problems connected with two typical interplanetary trips. The two trips were selected

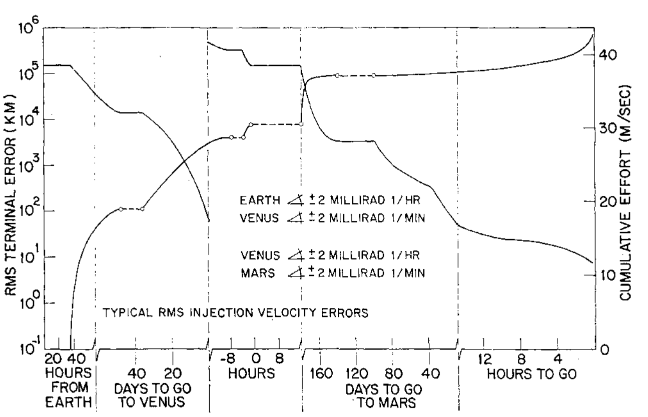

Fig. 4 History of remaining error and cumulative effort vs time to go (flyby trip, Earth-Venus-Mars).

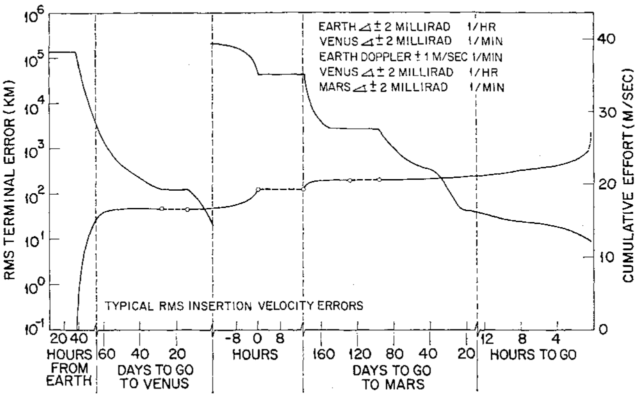

Fig. 5 History of remaining error and cumulative effort vs time to go (flyby trip, Earth-Venus-Mars).

from the results compiled by Lockheed.[6] They are: 1) a 204-day trip from Earth to Mars leaving Earth on December 30, 1966; and 2) a 245-day trip to Mars flying by Venus (seventy-five days to Venus and one hundred and seventy days to Mars). The trip leaves Earth on September 6, 1970.

The initial injection errors are assumed to be in velocity only. These errors can be obtained by propagating the errors at launch along the hyperbolic asymptote predetermined for this trip. Typical values of the errors at launch based on the Atlas-Agena booster were used. It turns out, after a simple computation, that for the 204-day trip to Mars, the 2×2 covariance matrix elements of the initial injection velocity error are of the order of 15–20 m/sec and are given specifically by

$$\begin{bmatrix} 330 & 107 \\ 107 & 130 \end{bmatrix} \left(\frac{\text{m}}{\text{sec}}\right)^2$$

The same injection errors were used for the flyby trip.

The following information rates were assumed: 1) angle information from the sun: ± 10 mrad at a rate of 1/hr; 2) angle information from Earth: ± 2 mrad at a rate of 1/hr; 3) angle information from Mars: ± 2 mrad at a rate of 1/min; 4) Doppler information from Earth: ± 1 m/sec at a rate of 1/min; 5) angle information from Venus (for flyby trip) ± 2 mrad at 1/min on first leg, and ± 2 mrad at 1/hr on the second leg. In all cases, the information from Earth was turned on after a wait of 3.6 hr so as to avoid an infinite information rate at the beginning.

The results of the 204-day trip to Mars are given in Figs. 1–3. In these figures, we have plotted the histories of the rms terminal miss and the average cumulative effort vs the time-to-go for different combination of measurement his-

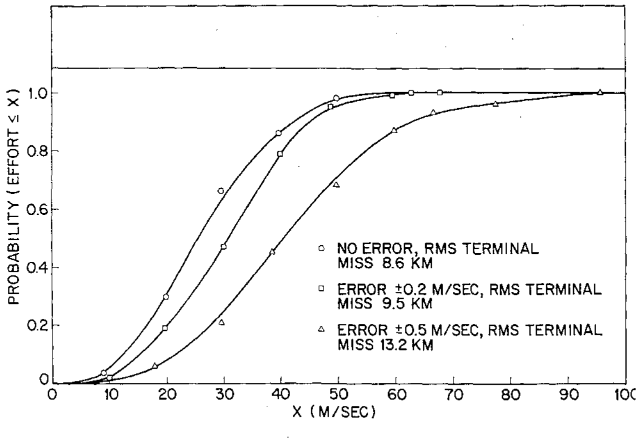

Fig. 6 Probability distribution of the cumulative effort

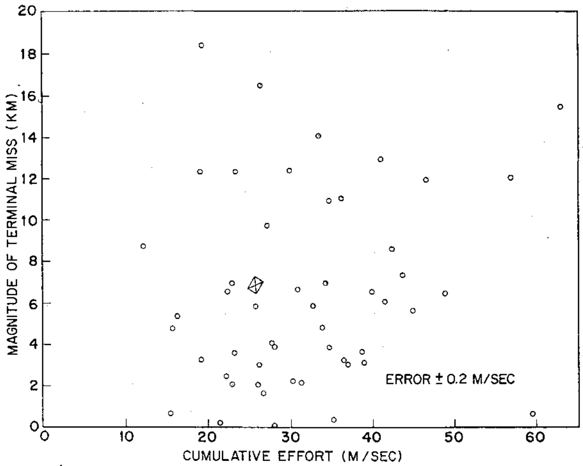

Fig. 7 Scatter diagram.

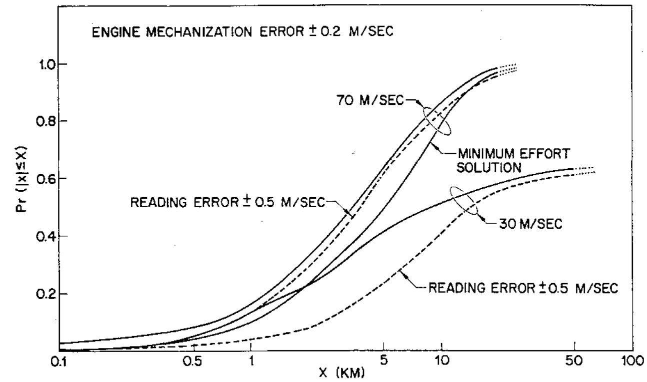

Fig. 8 Probability distribution of the magnitude of the terminal miss for fixed total velocity correction (the time of last correction is random).

tories. For the purpose of comparison, we have also plotted in Fig. 1 the corresponding near-optimum discrete strategy using four corrections based on the theory outlined in Sec. 3 (near-optimum in the sense that the timings are not optimized). It is seen that the correction effort required by the near-optimum discrete strategy is only about 10 to 15% higher than that required by the corresponding optimum continuous strategy.

The sudden drop in terminal miss in Fig. 2 is due to the impulsive thrust which is applied just prior to the time when the information from Earth vanishes. The control is turned on again very soon (of the order of a few hours) after the information from Mars is turned on. It is of interest to note that this is only a few percent more costly than the case of continuous observation (see Fig. 1). The effect of adding Earth range-rate information is shown in Fig. 3. It shows that most of the errors are corrected out at the beginning; and moreover, in the absence of any angle information from the planet, there exists an uncorrectable terminal miss which reflects our lack of knowledge of the actual error in the orbit estimation. This is of the order of 15 km as can be seen by the leveling off of the curves near the end. It is seen that this uncertainty can be eliminated by supplementing the measurements with angular information from Mars during the last forty days.

The results of the flyby trip are given in Figs. 4 and 5. Figure 4 shows the histories of the rms terminal miss and the average cumulative effort vs the time to go, for the case where only angle information is used. The Doppler is turned off when the vehicle reaches the end of the hyperbola. It is interesting to note that the corrective effort required for guidance on the flyby trip as far as Mars is not substantially greater than the effort required for the single leg trip to Mars.

Computer results for the Monte Carlo simulation of the discrete strategy shown in Fig. 1 are given in Figs. 6–8. Figures 6 and 7 correspond to four fixed correction times-and Fig. 8 corresponds to three fixed correction times and a randomized last correction. Figure 6 gives the empirical probability distribution (a sample size of 100) of the effort used for different rms engine execution errors and different values of rms terminal miss. It is assumed that the accelerometer reading error $\sigma_\alpha = 0$. A scatter diagram of the magnitude of the actual terminal miss and the cumulative effort for the case of $\sigma_\beta = 0.2$ m/sec is given in Fig. 7. The point marked ⊠ indicates the average effort and the average (absolute) terminal error under the assumption of no mechanization error. It is seen that a large number of points fall to the right of this ⊠, indicating that the amount of the fuel carried should be considerably in excess of the average amount used with fixed correction times. The results in Fig. 8 show an improvement in the terminal miss distribution, especially at the low end, as might have been anticipated. The same figure also shows the effect of the loss of information caused by an accelerometer reading error.

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